# Spatial Reuse and Fairness of Mobile Ad-Hoc Networks with Channel-Aware CSMA Protocols

Yuchul Kim, François Baccelli and Gustavo de Veciana

***Abstract*—We investigate the benefits of channel-aware (opportunistic) scheduling of transmissions in ad-hoc networks. The key challenge in optimizing the performance of such systems is finding a good compromise among three interdependent quantities, the density and channel quality of the scheduled transmitters, and the resulting interference at receivers. We propose two new channel-aware slotted CSMA protocols: opportunistic CSMA (O-CSMA) and quantile-based CSMA (Q-CSMA) and develop stochastic geometric models allowing us to quantify their performance in terms of spatial reuse and spatial fairness. When properly optimized these protocols offer substantial improvements in terms of both of these metrics relative to CSMA – particularly when the density of nodes is moderate to high. Moreover, we show that a simple version of Q-CSMA can achieve robust performance gains without requiring careful parameter optimization. The paper supports the case that the benefits associated with channel-aware scheduling in ad hoc networks, as in centralized base station scenarios, might far outweigh the associated overhead, and this can be done robustly using a Q-CSMA like protocol.**

## I. Introduction

The efficiency and fairness of a wireless ad-hoc network depends critically on how its associated Medium Access Control (MAC) protocol allocates shared resources, e.g., frequency, space, time, or codes. Starting with very simple protocols like ALOHA[1] used in the context of satellite-based communications, over the last decades, numerous approaches and protocols have been developed to enhance the operation of ad-hoc networks, culminating in the CSMA protocols used today. While there has been substantial research and development work on opportunistically exploiting channel variations for infrastructure-based, only a few works in the literature have specifically looked at this in context of *ad-hoc* networks – see [2], [12] and references therein, where an opportunistic variation of ALOHA is proposed and analyzed.

In this paper, we evaluate *channel-aware slotted CSMA protocols* for ad-hoc networks in terms of both *spatial reuse* and *spatial fairness*. We propose two MAC protocols, namely Opportunistic-CSMA (O-CSMA) and Quantile-based[1]-CSMA (Q-CSMA) that include two phases: channel-based qualification followed by contention resolution. O-CSMA is only opportunistic in the qualification phase where only the nodes having good channels to their receivers are qualified to contend. By contrast in Q-CSMA opportunism also plays a role in the contention process. In this paper we propose spatial stochastic geometric models for networks with randomly distributed nodes, that allow us to characterize the overall average performance of the network. We make the following key contributions.

1) We show that channel-aware CSMA protocols can improve *both* spatial reuse and fairness of ad-hoc networks over regular CSMA.
2) We characterize the subtle tradeoff between the density of active transmitters and the quality of transmissions as the function of qualification and carrier sense threshold and evaluate the spatial reuse performance of O/Q-CSMA.
3) We quantify spatial unfairness arising from the interactions between random nodes' locations and MAC protocols as the function of mean number of contending nodes. We show that, quantile-based opportunistic MACs can improve the fairness characteristics of CSMA networks.
4) We study the tradeoff between spatial fairness and reuse and compare the Pareto-frontier of O-CSMA and Q-CSMA. We show that the overall performance of Q-CSMA without the qualification step is as good as Q-CSMA and better than O-CSMA.

Our work is can be contrasted with previous work in following aspects. To our knowledge, this is the first attempt to consider the CSMA-based opportunistic MAC protocols in stochastic geometric framework. Second, this paper is the first to introduce fairness in the context of a stochastic network model, which is analytically tractable while capturing the impact of both the MAC and nodes' random placements. Most previous work[6], [11], [5] consider fairness for ad-hoc networks for a *fixed* graph which is quite revealing the impact of the underlying topology, but does not give a sense of the overall problem over an ensemble of node topologies.

The remainder of this paper is structured as follows. In Section II and III, we provide our models and metrics respectively. In Section IV, the performance of a typical node is analyzed as the function of system parameters. Based on that, spatial reuse and fairness are evaluated in Section V and VI respectively. We conclude in Section VII.

## II. System Model

### A. Node Distribution and Channel Model

We model the ad-hoc wireless network as a set of transmitters and their corresponding receivers. Transmitters are randomly distributed on the Euclidean plane as a homogeneous Marked Poisson Point Process (PPP) $\tilde{\Phi} = \{X_i, E_i, T_i, \mathbf{F}_i, \mathbf{F}'_i\}$, where $\Phi \equiv \{X_i\}_{i\geq 1}$ is a PPP with density $\lambda$ denoting the set of transmitters or their locations in $\mathbf{R}^2$ and $e_i$ is an indicator function which is equal to 1 if a node $X_i$ transmits and 0 otherwise, which is governed by the medium access protocol and surrounding nodes $\{X_j\}_{j\neq i}$. We assume that the receiver of each transmitter is $r$ meters away from the transmitter in random direction. Finally $\mathbf{F}_i = (F_{ij} : j)$ denotes a vector of random variables $F_{ij}$ denoting the fast fading channel gain between $i$th *transmitter* and the *receiver associated with $j$th transmitter*. We assume that $F_{ij}$s are symmetric, i.e., $F_{ij} = F_{ji}$ and independent and identically distributed with mean $\mu^{-1}$, i.e., $F_{ij} \sim F$, with cumulative distribution function (cdf) $G(x) = 1 - e^{-\mu x}$ with $x \geq 0$, which corresponds to the Rayleigh fading case.

We let $\|x\|$ be the norm of the vector $x \in \mathbf{R}^2$ and $l(\|x-y\|) = \|x-y\|^{\alpha}$ be the path loss (or slow fading) between two locations $x, y \in \mathbf{R}^2$ with a pathloss exponent $\alpha > 2$. Then, the

This work is supported by NSF grant NSF CNS-0917067.

[1]Similar approach for downlinks in cellular networks was introduced and studied in [10], [9] and for a wireless LAN setting in [7].

amount of interference power that the $j$th receiver at location $y$ experiences from the $i$th transmitter at location $x$ is given as $F_{ij}/l\left(\|x-y\|\right)$. The performance of the $i$-th receiver is governed by its signal to noise ratio given as $\mathrm{SINR}_i = \frac{F_{ii}/l(r)}{I_{\Phi\backslash\{X_i\}}+W}$, where $I_{\Phi\backslash\{X_i\}} = \sum_{X_j\in\Phi\backslash\{X_i\}} E_j F_{ji}/l\left(\|X_i - X_j\|\right)$ is the aggregate interference power from interferers, or so-called shot noise, and $W$ is thermal noise power. In interference limited networks, the impact of thermal noise is negligible as compared to interference, so in this paper it is ignored by letting $W=0$. Our assumption is that the $i$-th receiver gets $\log\left(1+t\right)$ bits per second (bps) per transmission if $\mathrm{SINR}_i > t$ and gets zero otherwise.

### B. Slotted Carrier Sense Multiple Access Protocols

The two MAC protocols we propose to study, O-CSMA and Q-CSAM, share two phases in the process of resolving which nodes will transmit.

**Qualification process**: We consider a *slotted* network, where only qualified nodes contend with their neighbors to access the medium. As in [2], [12], a node qualifies if its channel gain to its associated receiver exceeds a threshold $\gamma$. This requires that channel feedback from each receiver be available to its associated transmitter at each slot. Our model for this process is as follows. We let $\Phi^{\gamma} = \{X_i \in \Phi | F_{ii} > \gamma\}$ denote the set of qualified nodes or *contenders*. Because channel gains are assumed to be i.i.d., the point process of qualified nodes is a homogenous PPP corresponding to an independent thinning of the original PPP with probability $p_{\gamma} = \mathbb{P}(F > \gamma)$. Two transmitters $X_i$ and $X_j$ *contend* with each other if the received interference power they see from each other is exceeds a *carrier sense threshold* $\nu$, i.e., if $F'_{ij}/l(\|X_i - X_j\|) > \nu$ and by symmetry $F'_{ji}/l(\|X_i - X_j\|) > \nu$, where $F'_{ij} = F'_{ji} \sim F$ is the channel gain between two *transmitters* $X_i$ and $X_j$. The set of nodes contending with node $X_i$ will be called its *neighborhood* and denoted $\mathcal{N}_i^{\gamma} = \left\{X_j \in \Phi^{\gamma} : F'_{ji}/l(\|X_i - X_j\|) > \nu, j \neq i\right\}$. Clearly contending nodes should not be allowed to transmit simultaneously, which requires a contention resolution process amongst nodes in each neighborhood.

*Remark 1:* The qualification process is a mechanism to opportunistically select nodes currently experiencing high channel gains to their associated receivers. The posterior channel distribution for a node that has qualified is thus a shifted exponential denoted $G_{\gamma}(x) \equiv \mathbb{P}(F < x | F > \gamma) = (1 - e^{-\mu(x-\gamma)})\mathbf{1}_{\{x\geq\gamma\}}$, where $\mathbf{1}_{\{\cdot\}}$ is an indicator function. Note that the qualification process not only improves the transmit channel strength but *also* reduces the amount of interference. Unfortunately, we will see that parameter $\gamma$ needs to be chosen judiciously as it operationalizes a tradeoff between having a low density of contenders with very high quality channels but limiting the achievable spatial reuse versus a high density of nodes with lower quality channels possibly limiting the likelihood of successful transmissions.

**Contention Resolution**: The second phase resolves contention amongst contending nodes. A node contends with its neighbors, based on a timer value which is uniformly distributed on $[0,1]$. At the start of each time slot, a qualified node $X_j$ in $\Phi^{\gamma}$ starts its own timer and senses carrier. If it does not hear any node (in its neighborhood) prior to the expiration of its timer, it initiates transmission, otherwise it defers. O-CSMA and Q-CSMA differ in the way nodes generate their timer values. Note that timer values in practice need to be quantized, which in turn limits the performance of these protocols. Due to space constraints we will introduce the analysis of these effects but we refer the reader to [8].

*1) Opportunistic CSMA :* Under O-CSMA, a qualified node $X_i$'s timer value $T_i$ is simply a random variable uniformly distributed on $[0,1]$ at each slot and the node will transmit only if $T_i = \min_{j:X_j\in\mathcal{N}_i^{\gamma}\cup\{X_i\}} T_j$ i.e., it had the lowest timer value in its neighborhood.

*2) Quantile-based CSMA:* Under Q-CSMA, a qualified node $X_i$'s timer value $T_i = 1 - Q_i$ is tied to the randomness associated with channel gain variations to its receiver. Specifically the timer's value is related to the *quantile* $Q_i$ of the channel gain. Mathematically at each slot the channel gain quantile associated with a qualified node $X_j$ is $Q_j = G_{\gamma}\left(F_{jj}\right)$ where $G_{\gamma}(\cdot)$ is the cumulative distribution function for the channel gain of a node given it qualified, e.g., of $F_{jj}$ given $F_{jj} > \gamma$. The quantile and thus the timer value of a qualified node are still uniformly distributed on [0,1]. Yet the coupling between the timer and the channel introduces the announced opportunism. Under this mechanism, node $X_i$ transmits only if it has the lowest timer or *highest* quantile amongst the nodes in its neighborhood, i.e., when $Q_i = Q_i^{\max}$ where $Q_i^{\max} \equiv \max_{j:X_j\in\mathcal{N}_i^{\gamma}\cup\{X_i\}} Q_j$.

Under Q-CSMA the channel gain of an active transmitter, i.e., a node $X_i$ which qualified and won the contention resolution process resulting in $E_i = 1$, can be modeled as follows. The channel gain distribution for such a node is $F_{i,\gamma}^{\max} = G_{\gamma}^{-1}\left(Q_i^{\max}\right)$ where $G_{\gamma}^{-1}(\cdot)$ is the inverse function of $G_{\gamma}(\cdot)$. Letting $N_i^{\gamma} = |\mathcal{N}_i^{\gamma}|$ for simplicity, then $F_{i,\gamma}^{\max}$ is a $N_i^{\gamma}+1$th order statistic, i.e.,

$$F_{i,\gamma}^{\max} = \max\{F_{1,\gamma}, F_{2,\gamma}, \cdots, F_{N_i^{\gamma}+1,\gamma}\}, \tag{1}$$

with distribution $\mathbb{P}\left(F_{i,\gamma}^{\max} \leq x | N_i^{\gamma} = n\right) = (1 - e^{-\mu(x-\gamma)})^{n+1}\mathbf{1}_{\{x\geq\gamma\}}$ conditioned on $N_i^{\gamma} = n$.

*Remark 2:* Unlike O-CSMA, a Q-CSMA takes advantage of channel-awareness in both steps. Also one might expect Q-CSMA might work even better without a qualification phase since the quantile-based contention resolution can take advantage of opportunistic gain across a larger number of nodes in the neighborhood. We will see in the sequel that this insight is true only when the carrier sensing threshold $\nu$ is properly chosen. Yet the special case Q-CSMA without a qualification requirement, i.e., $\gamma = 0$ is of interest, and will be denoted $\mathrm{Q}_0$-CSMA.

### C. Further Notation

In the sequel we let $\mathcal{L}_I\left(s\right) = \mathbb{E}\left[e^{-sI}\right]$ denote the Laplace transform of the random variable $I$ and $\|x\|$ be the norm of $x \in \mathbf{R}^2$. We let $|C|$ denote the cardinality of set $C$ and let $\mathbf{R}_+$ denote the set of non-negative real numbers. For a point process $\Phi$ living in a set $\mathbf{N}$ and a set $\mathcal{Y} \subset \mathbf{N}$, following four probabilities denote the same quantity so-called Palm probability: $\mathbb{P}(\Phi\backslash\{0\} \in \mathcal{Y} | 0 \in \Phi) = \mathbb{P}^0(\Phi\backslash\{0\} \in \mathcal{Y}) = \mathbb{P}^{0!}(\Phi \in \mathcal{Y}) = \mathbb{P}(\Phi^0\backslash\{0\} \in \mathcal{Y})$, where we define $\Phi^0$ as a point process $\Phi$ given $0 \in \Phi$. For notational simplicity we will mainly use the second and fourth representations.

## III. Performance Measures

The two key performance metrics of interest are spatial reuse which measures how *efficiently* resources are reused by a given MAC protocol and spatial fairness which measures how *fairly* the space is used across nodes sharing the same space.

As a spatial reuse measure, we use the *density of successful transmissions* which is defined as the mean number of nodes that successfully transmit per square meter. This is given by

$$d_{suc} = \lambda p_{tx} p_{suc}, \tag{2}$$

where $\lambda$ is the density of transmitters, $p_{tx}$ is the transmission probability of a typical transmitter, and $p_{suc}$ is the transmission success probability of a typical receiver. Note that this metric not only measures the *level of spatial packing* through $\lambda p_{tx}$ but also measures the *quality of transmissions* through $p_{suc}$, which captures the interactions (through interference) among spatially packed nodes.

As a spatial fairness measure, we introduce a spatial version of Jain's fairness index which measures fairness based on long-term (or *time-averaged)* performance seen by nodes. Specifically, let $f_i(N_i, \mathbf{F}_i, \mathbf{F}'_i)$ be a performance metric of interest associated with node $X_i$, where $N_i$ is the number of other nodes in its neighborhood. Consider the random variable $\mathbb{E}\left[f_i(N_i, \mathbf{F}_i, \mathbf{F}'_i)|N_i\right]$ which captures variability in the mean performance seen by a typical node, conditioned on having neighborhoods of varying sizes. The spatial fairness measure proposed below is simply Jain's fairness index for this random variable, i.e., captures the degree to which the mean performance of nodes varies across nodes having neighborhoods of different sizes:

$$\tilde{\mathrm{FI}} = \frac{\left(\mathbb{E}^0\left[\mathbb{E}^0\left[f_0\left(N_0, \mathbf{F}_0, \mathbf{F}'_0\right)|N_0\right]\right]\right)^2}{\mathbb{E}^0\left[\left(\mathbb{E}^0\left[f_0\left(N_0, \mathbf{F}_0, \mathbf{F}'_0\right)|N_0\right)\right)^2\right]}, \tag{3}$$

where $\mathbb{E}^0$ denotes Palm expectation which is conditional expectation conditioned on a node at origin.

*Remark 3:* As explained in more detail in [8], this metric of fairness captures fairness in the mean performance seen across different classes of nodes, i.e., those which have different neighborhood sizes. This makes the metric analytically tractable, and still telling of the degree to which the protocol is able to rectify inherent network topology variations in the number of neighbors nodes will see.

## IV. Transmission Performance Analysis

In this section, we derive the transmission and success probability of a typical node for our two opportunistic protocols.

### A. Opportunistic CSMA

*1) Access Probability of a Typical Transmitter:* Under O-CSMA, we let $E_i = \mathbf{1}\{F_{ii} > \gamma, M_i < \min_{j:X_j \in \mathcal{N}_i^\gamma} M_j\}$ be the transmission indicator of $X_i$ and $\Phi_M^\gamma = \{X_i \in \Phi | E_i = 1\}$ be the set of active transmitters. Under Rayleigh fading, the transmission probability of a typical transmitter $X_0$ at origin is given by the probability that the node qualifies and gets the minimum timer value in its neighborhood, i.e., $p_{tx}^{op}(\gamma, \nu) = \mathbb{E}^0[E_0]$. Using the fact that the two events are independent and $N_0^\gamma = |\mathcal{N}_0^\gamma| \sim$ Poisson $(p_\gamma \bar{N}_0)$ where $p_\gamma = \mathbb{P}(F > \gamma)$ and $\bar{N}_0 = \mathbb{E}^0\left[\sum_{X_j \in \Phi \setminus \{0\}} \mathbf{1}\left\{F_{j0} > \nu l(|X_j|)\right\}\right]$, we get

$$p_{tx}^{op}(\gamma, \nu) = \mathbb{E}^0\left[\frac{p_\gamma}{1 + N_0^\gamma}\right] = \frac{1 - \exp\left\{-p_\gamma \bar{N}_0\right\}}{\bar{N}_0}. \tag{4}$$

Note that the case with $\gamma = 0$ (or $p_\gamma = 1$) corresponds to the pure CSMA scheme.

*2) Transmission Success Probability of a Typical Receiver:* Next we compute the transmission success probability of a typical receiver conditioned on its associated transmitter $X_0$ being at the origin, i.e.,

$$p_{suc}^{op}(t, \gamma, \nu) = \mathbb{P}^0\left(F > l(r) t I_{\Phi_M^\gamma \setminus \{0\}} | F > \gamma\right). \tag{5}$$

Note that the set of active transmitters $\Phi_M^\gamma$ is a point process induced by the qualification process followed by CSMA's contention resolution, which is known as the modified Matérn CSMA process [3]. Due to the interdependencies amongst node locations, it is hard to characterize this process. However, following [3], one can approximate $I_{\Phi_M^{\gamma 0} \setminus \{0\}}$ by an aggregate interference $I_{\Phi_h^{\gamma 0} \setminus \{0\}}$ in non-homogeneous PPP interferers with density, $\lambda^\gamma h(\tau, \lambda^\gamma)$ for $\tau > 0$, where $\lambda^\gamma \equiv p_\gamma \lambda$ and $h(\tau, \lambda^\gamma)$ is the conditional probability that a CSMA transmitter at distance $\tau$ from origin is active given an active CSMA transmitter at the origin and a density of qualified nodes $\lambda^\gamma$, see [8]. Since $h$ is a function which converges to 0 as $\tau \to 0$, and to $p_{tx}^{op}$ as $\tau \to \infty$, it captures the 'inhibiting' of transmitters realized by the CSMA MAC on $\Phi_M^{\gamma 0}$ from the perspective of an active transmitter at the origin.

For simplicity, let $F_\gamma$ is a random variable with the distribution function $G_\gamma$. Then, (5) can be approximated as follows by conditioning on the value of $F_\gamma$ and applying Plancherel-Parseval Theorem, see e.g., [4], [3]:

$$\int_{-\infty}^{\infty} \mathcal{L}_{I_{\Phi_h^{\gamma 0} \setminus \{0\}}}\left(2i\pi l\left(r\right) ts\right) \frac{\frac{\mu}{\mu - 2i\pi s} \exp\left\{2i\pi s \gamma\right\} - 1}{2i\pi s} ds. \tag{6}$$

A detailed derivation can be found in [8]. The next step is to compute the Laplace transform $\mathcal{L}_{I_{\Phi_h^{\gamma 0} \setminus \{0\}}}(s)$, which is given as

$$\mathcal{L}_{I_{\Phi_h^{\gamma 0} \setminus \{0\}}}(s) = \exp\left\{-\lambda^\gamma \int_0^\infty \int_0^{2\pi} \frac{h\left(\tau, \lambda^\gamma\right) \tau d\theta d\tau}{1 + \mu f\left(\tau, r, \theta\right)/s}\right\}, \tag{7}$$

where $f(\tau, r, \theta) = l\left(\sqrt{\tau^2 + r^2 - 2\tau r \cos\theta}\right)$. Substituting (7) into (6), we can numerically compute the approximate success probability $p_{suc}^{op}(t, \gamma, \nu)$.

### B. Quantile-based CSMA

*1) Access Probability of a Typical Transmitter:* Under Q-CSMA we have $E_i = \mathbf{1}\{F_{ii} > \gamma, Q_i > \max_{j:X_j \in \mathcal{N}_i^\gamma} Q_j\}$ of Q-CSMA node $X_i \in \Phi$. Then, using similar techniques as before, one can compute the access probability of a typical node $X_0$. Since the random variable $Q_i$ are independent and uniformly distributed, we have that $p_{tx}^{qt}(\gamma, \nu) = p_{tx}^{op}(\gamma, \nu)$.

*2) Transmission Success Probability of a Typical Receiver:* To determine the transmission success probability, we need to characterize the fading gain $F_{0,\gamma}^{\max}$ in (1) and the interference power $I_{\Phi_M^{\gamma 0} \setminus \{0\}}$ that a typical receiver sees. By contrast with the O-CSMA case, $F_{0,\gamma}^{\max}$ depends on $N_0^\gamma + 1$; in the sequel we make this explicit by writing $F_{0,\gamma}^{\max}(N_0^\gamma + 1)$. The transmission success probability to a typical receiver is thus given by

$$p_{suc}^{qt}(\gamma, t, \nu) = \mathbb{P}^0(F_{0,\gamma}^{\max}(N_0^\gamma + 1) > t l(r) I_{\Phi_M^\gamma \setminus \{0\}}). \tag{8}$$

Unlike (5), *the channel gain $F_{0,\gamma}^{\max}(N_0^\gamma + 1)$ is no longer independent of the aggregate interference $I_{\Phi_M^{\gamma 0} \setminus \{0\}}$*. To see this consider the following extreme cases. First, suppose $F_{0,\gamma}^{\max}(N_0^\gamma + 1)$ has a very small value close to 0, say $\epsilon$, then, this implies that the timer values of $X_0$'s neighbors are concentrated within the small interval $[1 - \epsilon, 1]$. Then, the neighbors of $X_0$'s neighbors are not likely to defer their transmissions to $X_0$'s neighbors. This means $X_0$'s receiver would experience a higher interference. While if $F_{0,\gamma}^{\max}(N_0^\gamma + 1)$ has a large value close to 1, say $\omega$, then, the timer values of $X_0$'s neighbors would be distributed in $[1 - \omega, 1]$, potentially resulting in deferrals of their own neighbors. In this case one might expect the interference level to be reduced. Thus both the channel gain $F_{0,\gamma}^{\max}(N_0^\gamma + 1)$ to, and interference level at the receiver depend on $N_0$. By conditioning on $N_0^\gamma$ and $F_{0,\gamma}^{\max}(N_0^\gamma+1)$, and approximating $I_{\Phi_M^{\gamma 0}\setminus\{0\}}$ for a given $N_0^\gamma=n$ and $F_{0,\gamma}^{\max}(N_0^\gamma+1)=x$ with an Interference $I_{\Phi_u^\gamma}$ from non-homogeneous Poisson interferers with density $\lambda^\gamma u(n,x,\tau,\lambda,\gamma)$, which is basically the conditional probability that a node $y_1$ transmits conditioned on following facts: 1) both $y_0$ and $y_1$ belong to $\Phi^\gamma$, 2) $y_1$ is a distance $\tau$ away from $y_0$, 3) $F_{0,\gamma}^{\max}(N_0^\gamma+1)=x$ or equivalently $y_0$'s timer value $T_0$ is given as $t_0=1-G_\gamma(x)$, 4) $N_0^\gamma=n$, and 5) $y_0$ transmits, i.e., $E_0=1$. It is written as

$$u(n,x,\tau,\lambda,\gamma)=\mathbb{P}(E_1=1|E_0=1,N_0^\gamma=n \\ ,F_{0,\gamma}^{\max}(N_0^\gamma+1)=x,\{y_0,y_1\}\subset\Phi^\gamma,|y_0-y_1|=\tau),$$

and given as (10), see [8] for the detailed derivation.

Then, applying the Plancherel-Parseval Theorem, (8) can be approximated as

$$\mathbb{E}^0\left[\int_{-\infty}^{\infty}\mathcal{L}_{I^{N_0^\gamma,F_{0,\gamma}^{\max}(N_0^\gamma+1)}_{\Phi_u^\gamma\setminus\{0\}}}(2i\pi l(r)ts)\frac{e^{2i\pi s F_{0,\gamma}^{\max}(N_0^\gamma+1)}-1}{2i\pi s}\mathrm{d}s\right],$$

where $I^{n,x}_{\Phi_u^{\gamma 0}\setminus\{0\}}$ is a random variable with cumulative distribution $\mathbb{P}^0(I_{\Phi_u^\gamma\setminus\{0\}}<z|N_0^\gamma=n,F_0^{\max}(N_0^\gamma+1)=x)$. Then, the Laplace transform $\mathcal{L}_{I^{N_0^\gamma,F_{0,\gamma}^{\max}(N_0^\gamma+1)}_{\Phi_u^\gamma\setminus\{0\}}}(s)$ is given by

$$\exp\left\{-\lambda^\gamma\int_0^\infty\int_0^{2\pi}\frac{u(N_0^\gamma,F_{0,\gamma}^{\max}(N_0^\gamma+1),\tau,\lambda^\gamma)\tau d\theta d\tau}{1+\mu f(\tau,r,\theta)/s}\right\}, \tag{9}$$

## V. Spatial Reuse

In this section, we first explore how $\lambda p_{tx}$ and $p_{suc}$ behave as the functions of the qualification threshold $\gamma$ and the carrier sensing threshold $\nu$, and then we compare the performance of O-CSMA and Q-CSMA. The results were numerically computed.

### A. Density of Active Transmitters $\lambda p_{tx}$

Fig. 1a exhibits the density of active transmitters $\lambda p_{tx}$ as a function $\lambda$. As $\lambda$ increases, the space is packed with a higher number of active transmitters, but it saturates to a value which we refer to a the asymptotic density of active transmitters defined as $\lambda_{csma}(\nu)\equiv\lim_{\lambda\to\infty}\lambda p_{tx}^{op}=\lim_{\lambda\to\infty}\lambda p_{tx}^{qt}$. It is easy to show that $\lambda_{csma}(\nu)=1/\hat{N}_0$, where $\hat{N}_0=\bar{N}_0^\gamma/\lambda^\gamma=\mathbb{E}[\int_{\mathbf{R}^2}\mathbf{1}\{F>\nu l(|x|)\}dx]$ is interpreted as the mean neighborhood size of a typical transmitter. Note that since each active transmitter occupy the area of size $\hat{N}_0$, intuitively, we can have at most $\hat{N}_0^{-1}$ active transmitters per unit space in the asymptotically dense networks. If $\gamma$ increases, the density of qualified transmitters $\lambda p_\gamma$ reduces, which accordingly decreases $\lambda p_{tx}$, but the converging value $\lambda_{csma}(\nu)$ is not affected. While if $\nu$ increases, the neighborhood size gets smaller, which allows a higher density of active transmitters, and accordingly higher $\lambda_{csma}(\nu)$.

### B. Success Probability of O-CSMA

Fig. 1b plots the success probability $p_{suc}^{op}$ as a function of $\lambda$ for various $\gamma$ and $\nu$ values. The general behavior of $p_{suc}^{op}$ is as follows. As $\lambda$ increases, $p_{suc}^{op}$ converges to a value between 0 and 1, since interference saturates. See [8] for details. If $\gamma$ increases, the signal quality at receivers increases and at the same time it reduces the density of active transmitters resulting in reduced interference power. Thus, increasing $\gamma$ boosts the SINR at receivers, and thus increases success probability. While if $\nu$ increases, the size of neighborhood is reduced and a higher number of active transmitters are allowed, which accordingly generates stronger aggregate interference, so both the received SINR and success probability decrease.

### C. Success Probability of Q-CSMA

Fig. 1c exhibits the success probability $p_{suc}^{qt}$ as the function of $\lambda$ for various $\gamma$ and $\nu$ values. The general behavior of $p_{suc}^{qt}$ is as follows. If $\lambda$ increases, $p_{suc}^{qt}$ decreases first due to the increased interference but soon converges 1 due to increasing opportunistic gain. If $\gamma$ increases, the interference level decreases due to the reduced density of active transmitters. However it is not clear if signal strength would show monotonically increasing behavior as was the case for O-CSMA, although it eventually increases as $\nu$ increases. This is because by increasing $\gamma$, the pdf of $F_\gamma^{\max}$ shifts to the right hand side (increasing the likelihood of success) but at the same time it decreases the size of neighborhood, thus the opportunistic gain coming from multiple contenders decreases (decreasing the likelihood of success). If $\nu$ increases, $p_{suc}^{qt}$ decreases due to the increased interference. Note that under the same parameter set, the success probability of Q-CSMA is always larger than O-CSMA, i.e., $p_{suc}^{qt}(t,\gamma,\nu,\lambda)\geq p_{suc}^{op}(t,\gamma,\nu,\lambda)$ simply due to the stochastic ordering relation : $F_\gamma^{\max}\geq^{st}F_\gamma$.

### D. Performance Comparison

Fig. 2a exhibits the density of successful transmissions of $Q_0$-CSMA and O-CSMA as the function of $\lambda$ for various values of $\gamma$ with $\nu=t=\mu=1$. As $\lambda$ increases, $d_{suc}^{op}(\gamma,\lambda)$ curves increase due to the increasing density of active transmitters, however they converge to some values since both $\lambda p_{tx}^{op}$ and $p_{suc}^{op}$ converge. For large $\gamma$, $p_{suc}^{op}$ is close to 1, so, as $\lambda$ gets large, $d_{suc}^{op}$ gets closer to the maximum performance that O-CSMA can achieve. When $\lambda$ is small, $d_{suc}^{op}$ decreases as $\gamma$ increases because the loss coming from decreased density of active transmitters is larger than the gain resulting from the increased quality of transmissions.

We note that the density of successful transmissions of Q-CSMA is always higher than that of O-CSMA, i.e., $d_{suc}^{qt}(t,\gamma,\nu,\lambda)\geq d_{suc}^{op}(t,\gamma,\nu,\lambda)$ for the same parameter set due to the fact that $p_{tx}^{qt}=p_{tx}^{op}$ and $p_{suc}^{qt}(t,\gamma,\nu,\lambda)\geq p_{suc}^{op}(t,\gamma,\nu,\lambda)$. What is more interesting here, is that the density of successful transmissions of $Q_0$-CSMA is better than that of O-CSMA, which implies the robustness of its performance to the density of nodes, see Fig. 2a. However, this is true only when the carrier sensing threshold $\nu$ is properly chosen. In this case, as $\lambda$ increases, the opportunistic gain from increasing number of neighbors is larger than the loss from increasing aggregate interference. If $\nu$ is large (inappropriate value), then this is not the case. Then, $Q_0$-CSMA will not be uniformly better than O-CSMA. For example, see the case $\nu=5$ in [8].

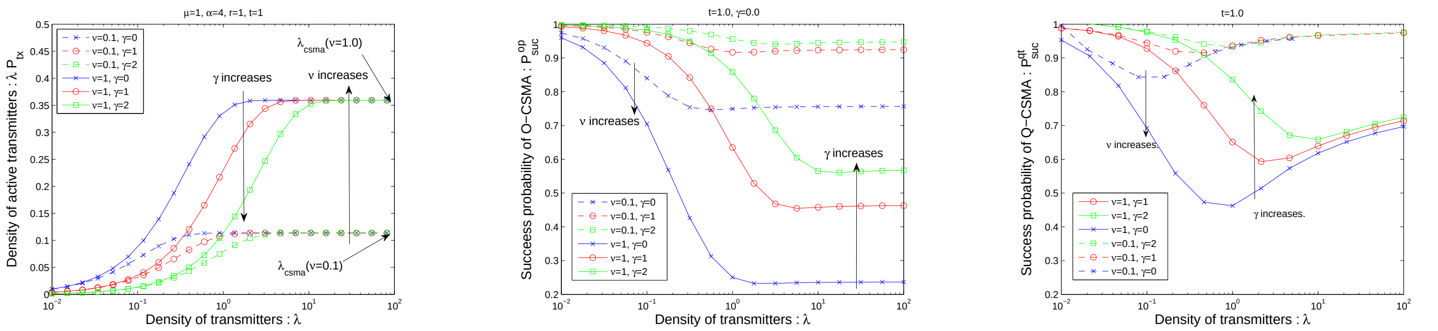
## VI. Spatial Fairness

In this section, we evaluate our spatial fairness metric and characterize the tradeoff between spatial reuse and fairness.

### A. Unfairness in CSMA Networks

It has been reported that (unslotted) CSMA networks are unfair [5], [11] due irregular network topologies and a combination of the carrier sense mechanism and binary exponential backoff. This can be partially mitigated by *slotting* since all nodes' contention

$$u(n,x,\tau,\lambda,\gamma) = \frac{\bar{N}_0^\gamma G(\nu l(\tau))}{n + (\bar{N}_0^\gamma - n) G(\nu l(\tau))} \Bigg( \frac{(1 - e^{-t_0 \bar{N}_0^\gamma (1-p_s)})}{\bar{N}_0^\gamma (1-p_s)} + $$
$$+ (1-t_0) e^{-\bar{N}_0^\gamma (1-p_s)} \sum_{k=0}^{n} \frac{k!}{\eta^{k+1}} \left( 1 - e^{-\eta} \sum_{j=0}^{k} \frac{\eta^j}{j!} \right) \binom{n}{k} p_s^k (1-p_s)^{n-k} \Bigg), \tag{10}$$




(a) The density of active transmitters for O/Q-CSMA increases and saturates as $\lambda$ increases due to the carrier sensing in CSMA protocol.

(b) The success probability of O-CSMA decreases as $\lambda$ increases, but converges to a value between 0 and 1.

(c) As $\lambda$ increases, the success probability of Q-CSMA decreases at first, but bounces and converges to 1 due to the increasing opportunistic gain.

Figure 1

windows are reset every slot. However, unfairness resulting from network topology irregularity remains. We will use (3) to quantify the fairness of O/Q-CSMA networks under the assumption that the fading between any two transmitters is averaged, i.e., $F'_{ij} = E[F] = \mu^{-1}$. Under this assumption, the channel gains from potential neighbors to a typical transmitter only depend on their pathloss, so the size of neighborhood does not change over time. This assumption is introduced to estimate the average fraction of time a node access the medium (access frequency). Let $N_{s,0}^\gamma$ be a random variable denoting the number of neighbors under this fading assumption with mean $\bar{N}_{s,0}^\gamma$.

## B. Fairness for access frequency and the Frequency of Successful Transmissions

Next we show that channel aware CSMA protocols has the potential to mitigate topological unfairness. We shall focus spatial fairness for access frequency and the frequency of successful transmissions.

For both O/Q-CSMA, we let $\mathbb{E}^0[f(N_{s,0}^\gamma, \mathbf{F}_0, \mathbf{F}'_0) | N_{s,0}^\gamma] = \frac{p_\gamma}{N_{s,0}^\gamma + 1}$ be the *access frequency* denoting the *fraction of time* a node with $N_{s,0}^\gamma$ neighbors can access medium. We let $\mathbb{E}^0[f(N_{s,0}^\gamma, \mathbf{F}_0, \mathbf{F}'_0) | N_{s,0}^\gamma] = \frac{1}{N_{s,0}^\gamma + 1} \bar{p}_{suc}^{op(qt)}(\gamma, N_{s,0}^\gamma)$ be the *frequency of successful transmissions* of a receiver, where $\bar{p}_{suc}^{op(qt)}(\gamma, N_{s,0}^\gamma)$ is the conditional success probability conditioned on that its associated transmitter has $N_{s,0}^\gamma$ contenders. $\bar{p}_{suc}^{qt}(\gamma, N_{s,0}^\gamma)$ is given as (11) and approximated as (12):

$$\mathbb{P}^0(F_{0,\gamma}^{\max}(N_{s,0}^\gamma + 1) > t I_{\Phi_{M \setminus \{0\}}} l(r) | N_{s,0}^\gamma = n) \tag{11}$$

$$\approx \mathbb{E}^0 \Bigg[ \int_{-\infty}^{\infty} \mathcal{L}_{I_{\Phi_u^\gamma \setminus \{0\}}^{N_{s,0}^\gamma, F_{0,\gamma}^{\max}(N_{s,0}^\gamma + 1)}} (2 i \pi l(r) t s) \times \tag{12}$$
$$\frac{e^{2 i \pi s F_{0,\gamma}^{\max}(N_{s,0}^\gamma + 1)} - 1}{2 i \pi s} ds \Big| N_{s,0}^\gamma = n \Bigg].$$

The corresponding spatial fairness index on access frequency across randomly distributed nodes is given as follows:

$$\tilde{\mathrm{FI}}_{ac}(\gamma, \bar{N}_{s,0}^\gamma) = \frac{\left( \mathbb{E}\left[ \frac{p_\gamma}{N_{s,0}^\gamma + 1} \right] \right)^2}{\mathbb{E}\left[ \left( \frac{p_\gamma}{N_{s,0}^\gamma + 1} \right)^2 \right]} = \frac{e^{\bar{N}_{s,0}^\gamma} + e^{-\bar{N}_{s,0}^\gamma} - 2}{\bar{N}_{s,0}^\gamma \left( Ei(\bar{N}_{s,0}^\gamma) - \log \bar{N}_{s,0}^\gamma - \eta \right)}, \tag{13}$$

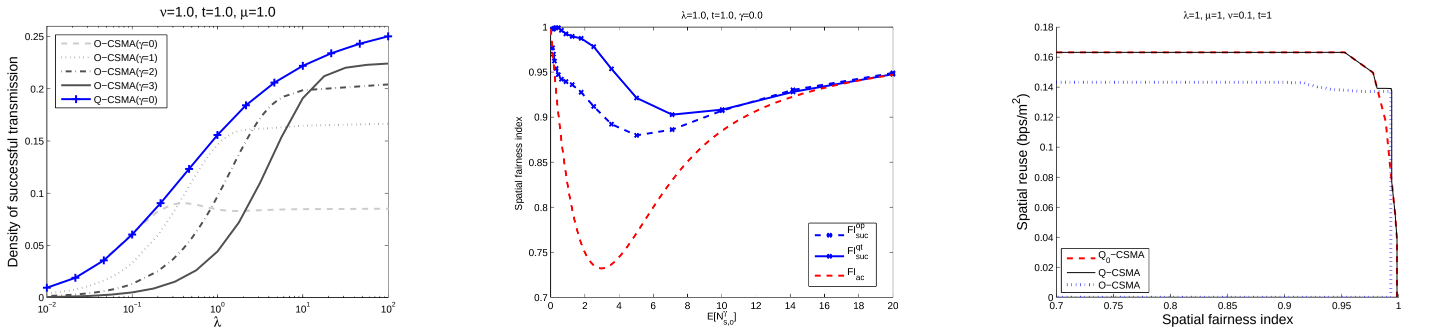
where $\bar{N}_{s,0}^\gamma = \mathbb{E}[N_{s,0}^\gamma]$, $Ei(x) = -\int_{-x}^{\infty} t^{-1} e^{-t} dt$ is an exponential integral function, and $\eta = 0.5772\ldots$. is the Euler-Mascheroni constant.

The corresponding fairness index on the frequency of successful transmission is given by

$$\tilde{\mathrm{FI}}_{suc}^{qt}(\gamma, \bar{N}_{s,0}^\gamma) = \frac{\left( \mathbb{E}^0\left[ \frac{p_\gamma}{N_{s,0}^\gamma + 1} \bar{p}_{suc}^{qt}(\gamma, N_{s,0}^\gamma) \right] \right)^2}{\mathbb{E}^0\left[ \left( \frac{p_\gamma}{N_{s,0}^\gamma + 1} \bar{p}_{suc}^{qt}(\gamma, N_{s,0}^\gamma) \right)^2 \right]}. \tag{14}$$

For O-CSMA, $\bar{p}_{suc}^{op}(\gamma, N_{s,0}^\gamma)$ can be approximated in a similar way and $\tilde{\mathrm{FI}}_{suc}^{op}$ can be defined accordingly. Using the fact that $N_{s,0}^\gamma \sim \mathrm{Poisson}(\bar{N}_{s,0}^\gamma)$, one can numerically compute $\tilde{\mathrm{FI}}_{suc}^{qt}(\gamma, \bar{N}_{s,0}^\gamma)$ and $\tilde{\mathrm{FI}}_{suc}^{op}(\gamma, \bar{N}_{s,0}^\gamma)$.

In Fig. 2b we plotted the $\tilde{\mathrm{FI}}_{ac}$ and $\tilde{\mathrm{FI}}_{suc}^{qt}$ and $\tilde{\mathrm{FI}}_{suc}^{op}$ for $\gamma = 0$. The dotted curve $\tilde{\mathrm{FI}}_{ac}$ denotes the fairness on access frequency for O/Q-CSMA versus $\bar{N}_{s,0}^\gamma\,(\nu)$. If $\bar{N}_{s,0}^\gamma$ is small, almost every node which contends gets to send, in fact all transmitters have access frequency close to $p_\gamma$, so fairness index is close to 1. If $\bar{N}_{s,0}^\gamma$ is relatively small, as $\bar{N}_{s,0}^\gamma$ (which is mean and the variability of the number of contenders) increases, the variability of access frequency, i.e., $\frac{p_\gamma}{N_{s,0}^\gamma + 1}$, across nodes increases resulting in a decrease in fairness. However, if $\bar{N}_{s,0}^\gamma$ is relatively large, the fairness index eventually increases again since, in this regime, the variability of access frequency $\frac{p_\gamma}{N_{s,0}^\gamma + 1}$ decreases and converges to 0, which in turn increases fairness. Note that the fairness curve has its minimum value $0.73019\ldots$, which corresponds to the minimum fairness index of slotted system. Specifically, the minimizer $n^* \equiv \arg\min_{n>0} \mathrm{FI}_{ac}(n) \approx 2.9736657$ can be found by numerically solving $\frac{\mathrm{d}}{\mathrm{d}n} \mathrm{FI}_{ac}(n) = 0$. Based on this, we make following argument. *The spatial fairness for access frequency of*




(a) Properly chosen $\nu$ both increases opportunistic gain of $Q_0$-CSMA and suppress the amount of aggregate interference for wide range of $\lambda$.

(b) Nodes with many neighbors have small fraction of access time. Q-CSMA compensates it with high success probability, accordingly fairness increases.

(c) The dominated set of $Q_0$-CSMA is as large as that of Q-CSMA and dominates the most of O-CSMA's dominated set.

Figure 2

*slotted O/Q-CSMA is worst, roughly 0.73 when the mean number of contenders of a typical transmitter is roughly 3.*

The figure also shows $\tilde{\mathrm{FI}}^{op}_{suc}$ and $\tilde{\mathrm{FI}}^{qt}_{suc}$ the fairness on the frequency of successful transmissions for O-CSMA and $Q_0$-CSMA respectively. Note that the $\tilde{\mathrm{FI}}^{op/qt}_{suc}$ is improved over $\tilde{\mathrm{FI}}_{ac}$, and $\tilde{\mathrm{FI}}^{qt}_{suc}$ is improved over $\tilde{\mathrm{FI}}^{op}_{suc}$. The gain is significant in the regime where $\bar{N}^{\gamma}_{s,0} \lesssim 10$. In this regime, the performance heterogeneity from different access frequency (due to random nodes placements) is high, but the increase of the success probability reduces the performance differences across nodes. In other words, the high success probability compensates the low access frequency, which decreases the variability of performance. While, in the regime where $\bar{N}^{\gamma}_{s,0}$ is large (or $\nu$ is small), the density of concurrent transmitters become small, which generates weak interference. Thus, most nodes succeed in transmission with high probability irrespective of the number of neighbors, so in this regime there is not much difference in performance. Thus, $Q_0$-CSMA and O-CSMA have similar performance.

So far, it has been shown that Q-CSMA can improve spatial fairness characteristics. However, with this result only, it is not clear how the density of successful transmissions and fairness jointly behave depending on system parameters. To better understand, we need to consider the pair of the performance measures.

### C. Tradeoff between Spatial Fairness and Spatial Reuse

In this section, we consider the tradeoff between spatial fairness and reuse under various parameter sets and the maximum performance that can be achieved. To that end, we make following definitions. We refer to $(a, b)$ with $a$ a fairness and $b$ the density of successful transmissions achievable under a given parameter setting, as an *FD-pair*. We say that an FD-pair $(a, b)$ *dominates* another $(c, d) \in \mathbf{R}^2_+$, if $a \geq c$ and $b \geq d$. We use $(c, d) \preceq (a, b)$ to denote this relation. We call the set $\Lambda(a, b) = \big\{(x, y) \in \mathbf{R}^2_+ | (x, y) \preceq (a, b)\big\}$ the *dominated set* by $(a, b)$, and the subset of FD-pairs which are not dominated by any other FD-pairs in the set is called as *Pareto-frontier* of the set. Using these definitions, we define the dominated set for O-CSMA, for a given $t$ and $\lambda$, as

$$\Omega^{op}(t, \lambda) = \bigcup_{\gamma \geq 0, \nu \geq 0} \Lambda(\tilde{\mathrm{FI}}^{op}_{suc}(t, \gamma, \nu, \lambda), d^{op}_{suc}(t, \gamma, \nu, \lambda)).$$

The dominated set of Q-CSMA $\Omega^{qt}(t, \lambda)$ is defined in a similar way, and that of $Q_0$-CSMA $\Omega^{qt}_0(t, \lambda)$ is defined as

$$\Omega^{qt}_0(t, \lambda) = \bigcup_{\nu \geq 0} \Lambda(\tilde{\mathrm{FI}}^{qt}_{suc}(t, 0, \nu, \lambda), d^{qt}_{suc}(t, 0, \nu, \lambda)).$$

Fig. 2c, plots $\Omega^{op}(t, \lambda)$, $\Omega^{qt}(t, \lambda)$ and $\Omega^{qt}_0(t, \lambda)$. As can be seen, the dominated set of $Q_0$-CSMA is very close to that of Q-CSMA, which makes $Q_0$-CSMA, *with one less parameter*, an attractive choice from an engineering point of view. There exists a subset of $\Omega^{op}(t, \lambda)$ which is not dominated by FD-pairs of $Q_0$-CSMA, however, this region is relatively small compared to the region of $\Omega^{qt}_0(t, \lambda)$ which is not dominated by O-CSMA.

## VII. Conclusion

In this paper, we showed that spatial reuse and fairness of CSMA ad-hoc networks can be significantly improved by using simple channel-aware CSMA protocols. In doing so, the optimal compromise between the density of active transmitters and the resulting aggregate interference needs to be made by controlling two system parameters : qualification and carrier sensing thresholds. We found that a simple version of Q-CSMA, with one less parameters, shows robust performance in both spatial reuse and fairness.

## References


[1] N. Abramson. The aloha system - another alternative for computer communication. *Proc. AFIPS*, pages 295–298, 1970.

[2] F. Baccelli and B. Blaszczyszyn. Stochastic analysis of spatial and opportunistic aloha. *IEEE Jour. Select. Areas in Comm.*, 27(7):1105–1119, Sep 2009.

[3] F. Baccelli and B. Blaszczyszyn. *Stochastic Geometry and Wireless Networks*, volume II. NOW Publisher, 2009.

[4] P. Brémaud. *Mathematical Principles of Signal Processing*. Springer, 2002.

[5] M. Durvy, O. Dousse, and P. Thiran. On the fairness of large csma networks. *IEEE Jour. Select. Areas in Comm.*, 27:1093–1104, September 2009.

[6] M. Durvy and P. Thiran. A packing approach to compare slotted and non-slotted medium access control. *IEEE INFOCOM*, pages 1–12, Apr 2006.

[7] C. Hwang and J.M. Cioffi. Using opportunistic csma/ca to achieve multi-user diversity in wireless lan. *IEEE Globcom*, pages 4952 – 4956, Nov 2007.

[8] Y. Kim, F. Baccelli, and G. de Veciana. Spatial reuse and fairness of mobile ad hoc networks with channel-aware csma protocols. *Technical Report*, 2011.

[9] D. Park, H. Kwon, and B. Lee. Wireless packet scheduling based on the cumulative distribution function of user transmission rates. *IEEE Trans. Comm.*, 53:1919 – 1929, Nov 2005.

[10] S. Patil and G. de Veciana. Measurement-based opportunistic scheduling for heterogenous wireless systems. *IEEE Transactions on Communications*, 57(9):2745–2753, Sep 2009.

[11] Xin Wang and K. Kar. Throughput modeling and fairness issues in csma/ca based ah-hoc networks. *IEEE INFOCOM*, 1:23–34, Mar 2005.

[12] S. P. Weber, J. G. Andrews, and N. Jindal. The effect of fading, channel inversion, and threshold scheduling on ad hoc networks. *IEEE Trans. Infomation Theory*, 53:4127–4149, Nov 2007.